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This grant was in its final no-cost extension during academic year 1994-5. Following the PI's move to Princeton Unversity, te remaining funds on the grant were administered under Cornell/Princeton Subcontract 20188-5423. They were to partially support the PI, Philip Holmes, during the academic year and summer 1995, and to support visiting facultiy P.S. Krishnaprasad (University of Maryland) and F. Verhulst (University of Utrecht, Holland), as well as providing some travel monies for the PI and Cornell students, R. Ghrist and H. Dankowicz.

Work Outlined in 1993-4 report continued in a number of areas. The final publication resulting form J. Duan's 1993 thesis appeared as did the paper with Dankowicz. That with Pratap is in press. These projects are now complete.

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Final Annual Technical Report on AFOSR 91-0329

NONLINEAR DYNAMICAL SYSTEMS IN MECHANICS AND BIOLOGY

Philip Holmes

Period 31 August 1994 - 15 July 1995 (No Cost Extension)

This grant was in its final no-cost extension phase during academic year 1994-5. Following the PI's move to Princeton University, the remaining funds on the grant were administered under Cornell/Princeton Subcontract 20188-5423. They were used to partially support the PI, Philip Holmes, during the academic year and summer 1995, and to support visiting faculty P.S. Krishnaprasad (University of Maryland) and F. Verhulst (University of Utrecht, Holland), as well as providing some travel monies for the PI and Cornell students, R. Ghrist and H. Dankowicz.

Work outlined in the 1993-4 report continued in a number of areas. The final publication resulting from J. Duan's 1993 thesis appeared [1.104], as did the paper with Dankowicz [1.103]. That with Pratap [1.106] is in press. These projects are now complete. Ongoing projects are outlined below.

1. Control of noisy heteroclinic cycles.

A third paper on controlling heteroclinic cycles has recently been submitted [1.111]. In it we extend the methods developed earlier to higher dimensional systems, which provide more realistic models of the turbulent boundary layer. We use optimal control theory in designing an algorithm to delay "bursting" (transit of heteroclinic cycles) in a fairly general class of symmetric systems. An announcement of preliminary results appeared in mid 1994 [2.86]. This project is now essentially complete. B. Coller has now graduated and will be a postdoctoral fellow at CalTech.

2. Knotted periodic orbits and bifurcation sequences.

Following [2.84], in the fall R. Ghrist and the PI discovered how to "infinitely renormalise" templates which carry the periodic orbits of certain

chaotic three dimensional flows. This provided a tool which enabled the construction of a universal template: a template which simultaneously supports all posssible knots and links! One of the most dramatic implications is that we can display "simple" three dimensional ODEs which contain, among their periodic orbits, all knots and links [2.90]. R. Ghrist has graduated and will be an NSF postdoctoral fellow at the Institute for Advanced Study and the University of Texas, Austin.

3. Homoclinic Bifurcation in a three dimensional family of flows.

A. Doelman and the PI completed an extensive paper [1.107], in which we analyse in detail the interaction between simple codimension one bifurcations, a local saddle-node and a global homoclinic loop, in a perturbed Hamiltonian system. We find infinite cascades of heteroclinic and homoclinic bifurcations, implying the existence of complex travelling wave patterns in the Ginsburg Landau PDEs from which systems of this type derive. A revised version has recently been accepted and is currently in press.

4. Wavelets and local models.

With the help of a NATO travel grant, G. Berkooz, J. Elezgaray and the PI have continued using wavelet projections to extract spatially localised ODE models of evolution equations from large domains [2.75]. With Mark Myers (former Applied Math Grad student of John Guckenheimer) we completed a study of the symmetry breaking due to wavelet projections of a translation-reflection symmetric system: the Kuramoto-Sivashinsky equation [1.108]. We also showed how wavelets provide a natural "language" in which to carry out renormalisation type calculations [1.100]. H. Dankowicz worked on a related project, studying modal interactions in Fourier projections of the same equation with constant and time varying lengths, representing the varying boundaries of a small subsystem in a larger spatio-temporally chaotic field. We find that, for suitable broad choices of (constant) lengths, good short time tracking can be achieved, but that long term statistics such as spatial power spectra are more sensitive to length choices. A paper describing this work has recently been revised and is in review [1.109]. H Dankowicz has graduated and is currently a postdoctoral fellow at the Royal Technological Institute, Stockholm.

4. Coupled Oscillators.

A first year PhD student, David Taylor, has begun working with the PI on coupled oscillator models with a view to obtaining a more detailed understanding of recent experiments on lamprey central pattern generators (CPGs) with attached brainstem, due to Avis Cohen of the University of Maryland. The model incorporates "internal structure" - the individual oscillators can bifurcate from excitable, with stable and unstable equilibria, to bursting, with a stable limit cycle. We have already discovered that, under uniform external stimulation, the same coupling architecture and strengths can yield either forward- or backward-going waves in a linear chain. Some of this work is described in [2.89]. We are currently studing a simple 2 oscillator model - a flow on a two torus - and finding a rich panoply of local and global (heteroclinic) bifurcations. We intend to use the results of these studies to "design" oscillator networks for locomotion generation in robotics as well as apply them to understanding CPGs. P. Krishnaprasad is involved in this ongoing project, which began during his visit to Princeton.

Honors, Invitations and Awards

The PI will lecture in the NATO/ASI "From Finite to Infinite Dimensional Dynamical Systems," to be held at the Newton Institute, Cambridge, UK, 21 August-1 September, 1995. He will also give invited lectures in the research conference to follow and will participate in the parent 6-month research program at the Newton Institute. The PI has been invited to deliver a plenary address at the Winter Annual AMS Meeting in Orlando, FL, Jan 10-13, 1996. He will be the Lansdowne Visiting Lecturer at the University of Victoria, BC, Canada, in March 1996. Finally, the PI wrote an invited article [2.93] based on a public leture given during his 1987 tenure of the Aisenstadt Chair at the Centre de Recherches Mathématique, Montréal, to appear in the CRM 25th Anniversary volume.

Publications

[1.103] H. Dankowicz and P. Holmes (1993) *J. Diff. Eqns 116(2), 468-483.* The existence of transverse homoclinic points in the Sitnikov problem.

[1.104] J. Duan and P. Holmes (1995) *Proc. Edinburgh Math. Soc. 38, 77-97.* Fronts, domain walls and pulses in a generalized Ginzburg-Landau equation.

[1.105] B.D. Coller, P. Holmes and J.L. Lumley (1994) *Phys. Fluids 6(2), 954-961.* Interaction of adjacent bursts in the wall region.

[1.106] R. Pratap and P. Holmes (1994) *Nonlinear Dynamics (in press).* Chaos in a mapping describing elasto-plastic oscillations.

[1.107] A. Doelman and P. Holmes (1994) *Phil. Trans. Roy. Soc. London (to appear).* Homoclinic explosions and implosions.

[1.108] M. Myers, P. Holmes, J. Elezgaray and G. Berkooz (1994) *Physica D (in press).* Wavelet projections of the Kuramoto-Sivashinsky equation I: heteroclinic cycles and modulated travelling waves for short systems.

[1.109] H. Dankowicz, P. Holmes, J. Elezgaray and G. Berkooz (1995) *Physica D (submitted).* Local models of spatio-temporally complex fields.

[1.110] J. Elezgaray, G. Berkooz and P. Holmes, (1995) *Phys. Rev Lett. (submitted).* Large scale statistics of the Kuramoto-Sivashinsky equation: a wavelet based approach.

[1.111] B.D. Coller and P. Holmes, (1995) *Automatica (submitted).* Suppression of bursting.

[2.75] J. Elezgaray, G. Berkooz and P. Holmes (1993) Wavelet analysis of the motion of coherent structures. In "Progress in Wavelet Analysis and Applications", 471-476, Ed. Y. Meyer and S. Roques, Editions Frontières,

Gif-sur-Yvette, 1993.

[2.84] P. Holmes and R. Ghrist (1993) Knotting within the gluing bifurcation. IUTAM Symposium on Nonlinearity and Chaos in Engineering Dynamics, University College, London, July 19-23, 1993. To appear in "Nonlinearity and Chaos in Engineering Mechanics", ed. J.M.T. Thompson and S.R. Bishop, Wiley, Chichester, U.K.

[2.86] B.D. Coller, P. Holmes and J.L. Lumley (1994) Control of bursting in boundary layer models. 12th U.S. National Congress of Applied Mechanics, Seattle, WA, June 26-July 1, 1994. In "Mechanics USA 1994", ed. A.S. Kobayashi, *Appl. Mech. Rev.* 47 (6), part 2, S139-S143.

[2.89] D.A. Taylor, P. Holmes and A.H. Cohen (1995) Excitable oscillators as models of central pattern generators. To appear in "Nonlinear Dynamics: The Richard Rand 50th Birthday Volume", A. Guran, editor.

[2.90] R. Ghrist and P. Holmes (1995) An ODE whose solutions contain all knots and links. To appear in a special issue of *Int. J. of Bifurcation and Chaos* celebrating the 60th birthday of L.P. Shilnikov, L. Lerman, editor.

[2.93] P. Holmes (1995) Metaphor and Models in Science and Art. To appear in a special volume celebrating the 25th Anniversary of the Centre de Recherches Mathématiques, Université de Montréal, *CRM Proceedings and Lecture Notes*, CRM/AMS Presses, Montréal, Quebec and Providence, RI.